What Was on Top of Archimedes' Tomb?

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At The High School Math Contest
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Attention iPad Users!

The iPad renders three slides incorrectly —

The ones with captions beginning:

“Cylinder”, “Cone”, and “Sphere”.

(Verified on the date of the talk)

Archimedes:

He invented physical modeling

and the mathematics needed to do it!

Our example: How he used the law of the lever to discover the volume of the sphere.

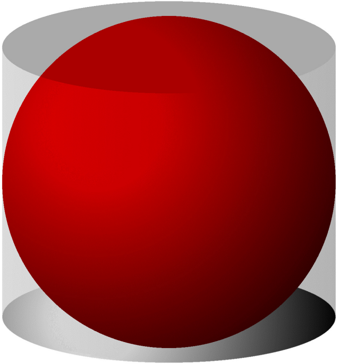
Little is known about him.

- Archimedes of Syracuse, 287 ? – 212 BC
- Greek mathematician, physicist, engineer, inventor and astronomer
- Approximated π, determined the area of a circle and the volume of a sphere in terms of π
- Invented the compound pulley and *explained* the mechanical advantage of the lever
- Laid foundations in hydrostatics and statics, calculated area of parabola using summation of an infinite series, and defined the spiral of Archimedes
- Killed by a Roman soldier during the capture of Syracuse.

Archimedes and the Roman Soldier

(Anon.)

Archimedes' Proudest Achievement



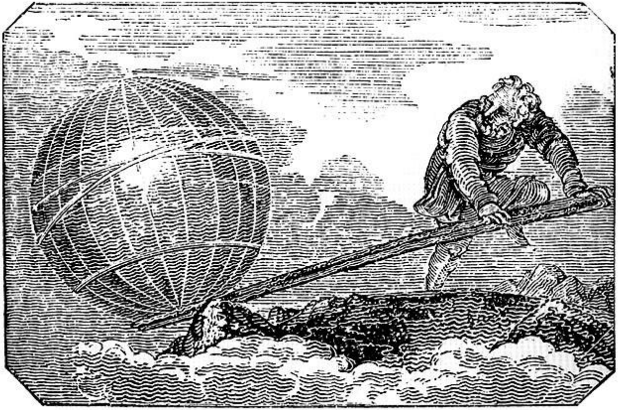
(NYU, http://math.nyu.edu/ crorres/Archimedes/Tomb/Cicero.html)

The enclosed sphere has $2/3$ the volume of the cylinder.
In this talk we begin with the Law of the Lever, then conclude with Archimedes' use of it to determine the volume of a sphere.

Part 1

The Law of the Lever

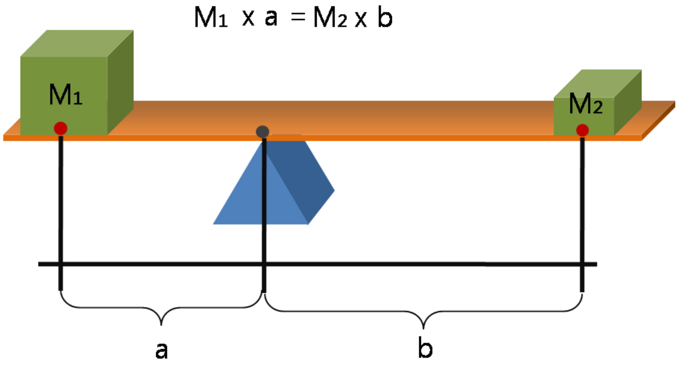
Archimedes' Moves the World



(Anon.)

"Give me a lever long enough and a place to stand and I will move the world."

The Law of the Lever

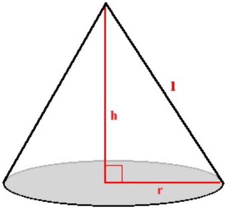


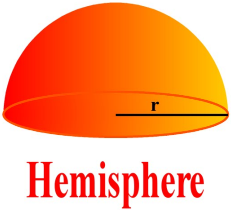
(Wikipedia)

Archimedes assumed: A mass presses down on a static beam as if concentrated at its *center-of-mass*.

The beam is stiff and weightless.

Archimedes invented the Center-of-Mass.





(Clip Art Freeware)

Where is the center-of-mass? This usually needs calculus, but Archimedes understood the concept without *modern* calculus.

Archimedes' Innovations

Archimedes:

1. invented the concept of center-of-mass;
2. introduced a rigorous mathematical model to describe a physical phenomenon: the Law of the Lever;
3. applied his law of the lever to find the volume of a sphere — this was the mysterious Mechanical Method he referred to in his correspondence with other geometers

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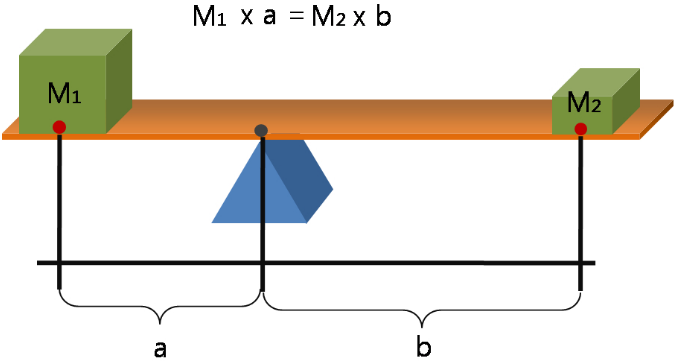
The Law of the Lever



(Wikipedia)

Part II

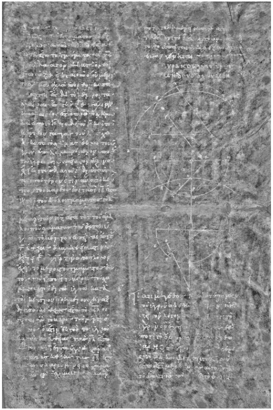
Volume of the Sphere

Archimedes' Method, unknown until 1906, applies the Law of the Lever.

Archimedes explained his "Mechanical Method" in a Palimpsest discovered in 1906 in a Byzantine Crypt in Istanbul.

Here's what it looked like when rediscovered in 1998.

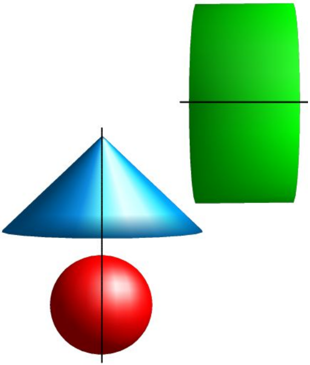
One Imaged Page of the Archimedes Palimpsest



(Wikipedia)

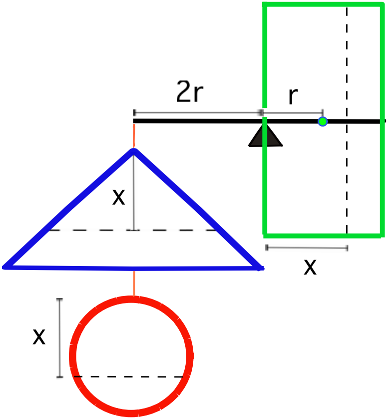
In the early 1900s, the Danish philologist Johan Heiberg transcribed legible portions into Greek. Here is a recent scan.

Archimedes used the Law of the Lever to compare volumes of a cylinder, cone and sphere.



Archimedes knew the volumes of cylinders and cones. and areas of their circular cross sections. He used these to determine the volume of the sphere.

Here's how he placed them.



The objects have equal uniform density.

Radius of sphere $= r$, base radius and height of cone $= 2r$.

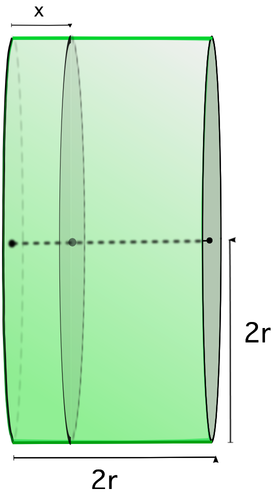
Archimdedes compared the x-level cross-sectional areas.

Cylinder: Area of x-level Cross Section

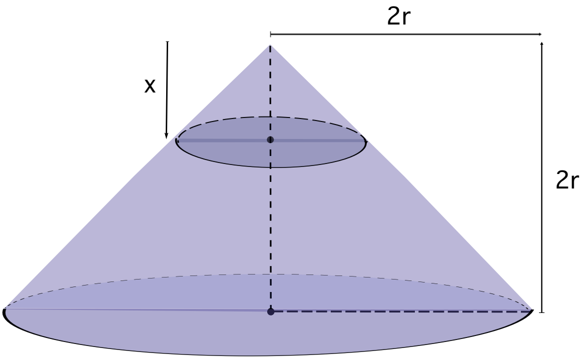
Let r be the radius of the sphere.

Circular cross-section at level x: $4\pi r^2$

[Volume of cylinder = $C_y = 8\pi r^3$]



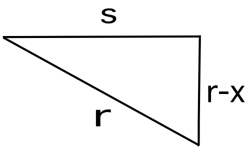
Cone: Area of x-level Cross Section

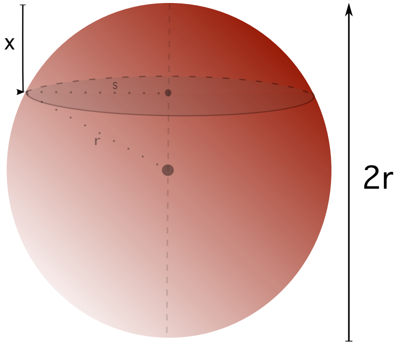


Circular cross-section at level x: πx^2

[Volume of cone = $C_o = \frac{8}{3}\pi r^3$]

Sphere: Area of x-level Cross Section

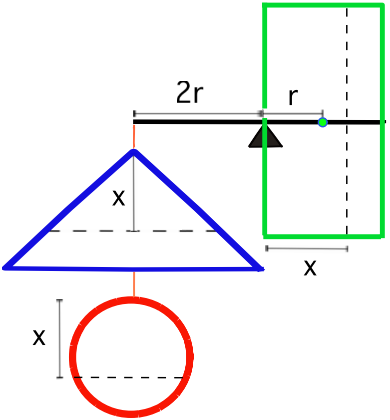




Circular cross-section at level x: $\pi s^2 = 2\pi r x - \pi x^2$

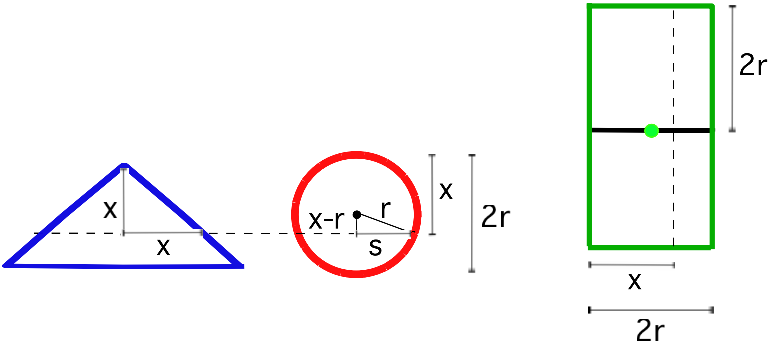
[Volume of sphere = $S =???$]

Method (Step 1): Three Solids in Perfect Balance



We will see that the *x*-level cross sections are in balance.

Method (Step 2): The cross-section (CS) areas balance.



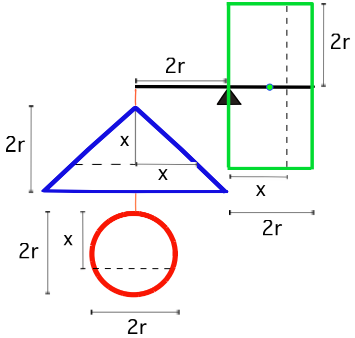
Cone CS: $+\pi x^2$;

Sphere CS: $\pi s^2 = 2\pi r x - \pi x^2$;

(Cone CS + Sphere CS) = $2\pi r x$ Cylinder CS: $4\pi r^2$

$2r \cdot$ (Cone CS + Sphere CS) = $4\pi r^2 x$ = Cylinder CS $\cdot$ x **QED**

Method (Step 3): The Derivation



By previous slide: Each x-section of cylinder balances the combined x-sections of the hanging cone and sphere. Therefore, **the volumes balance**: $2r \cdot (C_o + S) = r \cdot C_y$, so

$$C_o = \frac{8}{3}\pi r^3, \quad C_y = 8\pi r^3 \quad \Longrightarrow \quad S = \frac{4}{3}\pi r^3$$

Using modern terms, we would express this as:

$$2r(C_o + S) =$$

$$\int_0^{2r}(2r)(2\pi rx)dx = 8\pi r^4 = \int_0^{2r}(x)(4\pi r^2)dx$$

$$= rC_y$$

Archimedes did this nearly 2 millenia before Newton and Leibniz.

What was on top of his Tomb?

Archimedes asked that a cylinder and sphere be mounted on his tomb, displaying their proportional volumes.

The sphere is $2/3$ the volume of a circumscribed cylinder

(because Sphere$= \frac{4}{3}\pi r^3$, and Cylinder$= 2\pi r^3$)

And so it was done, as the Roman Orator Cicero discovered 137 years after the death of Archimedes.

"Cicero discovering the Tomb of Archimedes – 1"

(Anon., 1806)

Frontispiece of Weinzierl's German translation of Cicero's *Tusculan Disputations*

"Cicero discovering the Tomb of Archimedes – 2"

(Martin Knoller (1725-1804))

"Cicero discovering the Tomb of Archimedes – 3"

(Benjamin West (1738-1820))

"Cicero discovering the Tomb of Archimedes – 4"

(Hubert Robert (1733-1808))

"Ciceron Decouvrant le Tombeau d'Archimede" – 5

(Pierre Henri de Valenciennes (1750-1819))

Conclusion

Archimedes invented physical modeling, using rigorous mathematical deductions from specified physical axioms.

He formulated and *proved* the Law of the Lever, based on center-of-mass.

He anticipated methods of integral calculus: Cavalieri's Principle and Fubini's theorem.

And now we know his Mechanical Method, by which he applied the Law of the Lever to determine the volume of a sphere.

Recommended reading

- Asger Aaboe, *Episodes from the early history of mathematics*, Mathematical Association of America, 1998.
- Archimedes, *The works of Archimedes*, Dover Publications, (Thomas L. Heath edition), 2002.
- E. J. Dijksterhuis, *Archimedes*, Acta Historica Scientiarum Naturalium et Medicinalium, vol 12, 1956.
- Laubenbacher and Pengelley, *Mathematical Expeditions — Chronicles by the Explorers*, Springer, 1999.
- Sherman Stein, *Archimedes: What Did He Do Besides Cry Eureka?* Mathematical Association of America, 1999.

- Nietz & Noel, *The Archimedes Codex: How a Medieval Prayer Book Is Revealing the True Genius of Antiquity's Greatest Scientist*, Da Capo Press, Jan 9, 2009.
- The Archimedes Palimpsest ⟨`http://www.archimedespalimpsest.org/`⟩

A more detailed version of this talk (with proofs of the Law of the Lever) was given at UNM on Feb 4, 2012.

These slides continue with brief comments.

These slides and the UNM talk are on my web.

www.mikeraugh.org

Contact me at,

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(Supplementary sections follow.)

Brief Comments

Quality of Archimedes' Work

"In weightiness of matter and elegance of style, no classsical mathematics treatise surpasses the works of Archimedes. This was recognized in antiquity; thus Plutarch says of Archimedes' works:

> *'It is not possible to find in all geometry more difficult and intricate questions, or more simple and lucid explanations. Some ascribe this to his genius; while others think that incredible effort and toil produced these, to all appearances, easy and unlaboured results.' "*

(Aaboe in *Recommended reading*)

"Archimedes is so clever that sometimes I think that if you want an example of someone brought from outer space it would be Archimedes. Because he, in my view, is so original and so imaginative that I think he is better than Newton. Whereas Newton said, 'I have only seen so far because I have been standing on the shoulders of other giants,' there was nobody for Archimedes, nobody's shoulders for Archimedes to stand on. He is the first physicist and the first applied mathematician. And he did it all on his own from nowhere."

(Lewis Wolpert in *On Shoulders of Giants* by Melvyn Bragg, 1998)

Archimedes' "Method of Mechanical Theorems."

Archimedes' geometric proof for the volume of a sphere was well known, but the method by which he discovered the result remained a mystery until 1906.

The *Archimedes Palimpsest*, in which Archimedes described his method, was found in Istanbul in a Byzantine crypt, then lost and recovered again in 1998.

The Palimpsest contained a tenth-century copy of a Greek MS that was scraped, washed and overlaid with Christian liturgy and other writings.

(See *Recommended reading*).

The Roman orator Cicero found the Archimedes' tomb.

Archimedes was killed in 212 BC.

Archimedes had asked that a cylinder and sphere be mounted on his tomb, displaying their proportional volumes: 2/3.

The tomb was built and lost until the figures of the cylinder and the sphere enabled Cicero to find it 137 years later.

Cicero found the tomb in 75 BC. He wrote:

"So one of the most famous cities in the Greek world would have remained in total ignorance of the tomb of the most brilliant citizen it had ever produced, had a man from Arpinum not come and pointed it out!"

(Aaboe in *Recommended reading*)