

How do you know what time it is?
The difference between clock time and solar time

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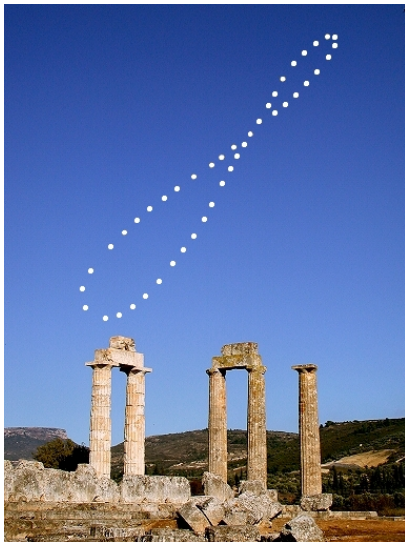
Let no one ignorant of geometry enter my Academy!

—Plato

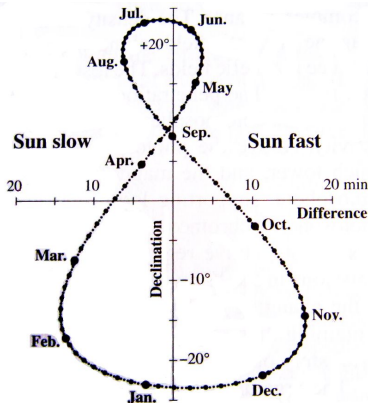
Time is our topic

- Our sense of time is governed by the Sun:
Hours, Seasons, even moonlight giving us the months.
- This talk explains the difference between *solar time* (“sundial time”) and *clock time*—a problem in geometry.
- The next image of a *solar analemma* illustrates the difference.
- It's a time lapse image of the Sun, made over a one-year period from a fixed location at a fixed clock time.

Time lapse photo of Sun at the Temple of Zeus!



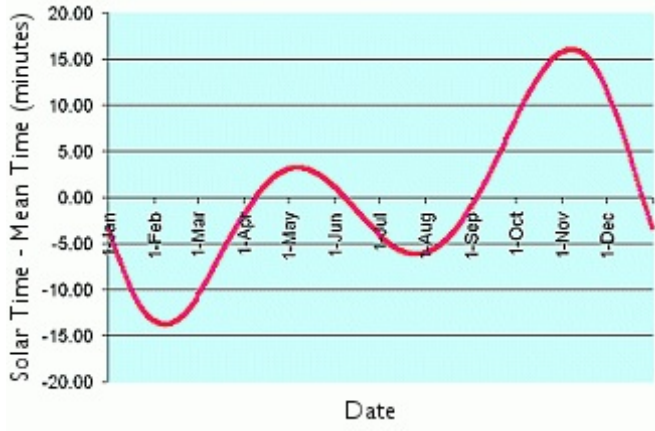
Schematic of the Solar Analemma



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When your watch (set for standard "mean" time in your time zone) strikes noon, the graph shows how far in minutes the Sun is from crossing the *standard meridian* in your time zone—explained below.

The schematic unwrapped yields the “Equation of Time”



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The next slide explains how to use the Equation of Time.

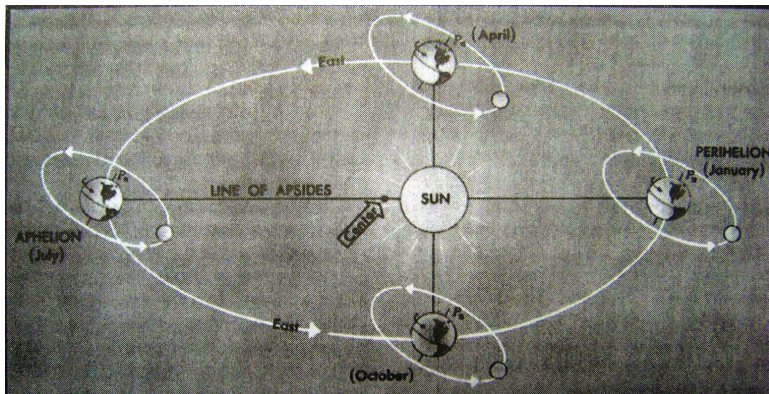
How to use the Equation of Time

The solar analemma shows that the Sun moves through the sky at a pace that varies throughout the year, so it is not suitable for synchronizing time in today's world. *Mean time* refers to a way of pacing time evenly so that each day of the year holds the same number of minutes, seconds and hours, permitting clocks with hands that move at an even pace to be used for telling time instead of the varying Sun.

There are 24 *time zones* circling the globe, each of width 15° longitude, with some exceptional zig-zagging around islands and political boundaries. At the center of each zone is its *Standard Meridian*. (A *meridian* is half a great circle terminating at the North and South poles of the Earth's axis.) The *Prime Meridian* at 0° longitude passes through the Greenwich Observatory in England.

The standard for mean time is set at the Greenwich Observatory, and the standard mean time for each time zone west of Greenwich is progressively one hour earlier. For example, the standard meridian for Los Angeles—the meridian for Pacific Standard Time—is at 120° west longitude, where the mean time is eight hours earlier than the mean time at Greenwich. To interpret the Equation of Time, imagine yourself positioned in LA at the 120^{th} meridian on November 1. Noting that November is on the “Sun fast” side of the Schematic shown on a previous slide, the Equation of Time tells you that when your watch (set for Pacific Standard Time) strikes 12:00 noon, the Sun will have already passed its high point in the sky about sixteen minutes earlier.

Cause of the solar analemma



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The analemma and the Equation of Time are due to: (1) the varying rate of the Earth in its elliptical orbit around the Sun, and (2) the tilt of the Earth's axis with respect to the plane of the Earth's orbit. This figure nicely illustrates these factors, but there's a better way to model the relationship of the Earth and the Sun, as we shall see.

Telling time by the Sun: The Larkin Sundial

The next slides show photos of the *bowstring equatorial sundial* at Ralph B. Larkin Memorial Park in Claremont, California (within two degrees east of the Pacific Standard Time Meridian). The “bowstring” is aligned with the Earth’s axis ensuring that the shadow of the bowstring moves across the circular “equatorial” scale at the same uniform rate as the Earth’s rotation. The equatorial scale is adjusted to compensate for the geographic offset of the sundial from the Pacific Standard Time Meridian.

A special feature of the Larkin sundial is the analemma-shaped *gnomon* suspended by the bowstring, which adjusts the shadow to correct for Pacific Standard Time. Standard time as told by the Larkin sundial is accurate to within five minutes throughout the year.

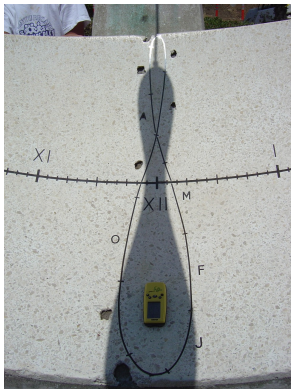
The Larkin Memorial Sundial



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The photo was taken at noon Pacific Standard Time on September 1. The bowstring's shadow falls slightly askew on the "meridian" (frame) of the sundial because Larkin Park lies near but not on the Pacific Standard Meridian. The next slide shows that the noon mark ("XII") is shifted off center slightly to compensate for this geographic offset.

Time according to the Larkin Memorial Sundial



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The shadow of the gnomon drops in summer and rises in winter, placing the appropriate portion of the analemma on the time scale. The initials for the months tell you which side of the shadow to read. The initials for months January, February, March, April, and October are still intact, others have fallen out with decay.

What makes it work? Begin with “rotation of the stars”!

- The Stars seem fixed to a rotating Celestial Sphere.
- The Greeks thought the Earth was fixed—that the stars rotate around the Earth and that the Sun moves among the stars.
- Wrong, but not a bad idea considering the evidence!
- And convenient conceptually....

“Star Trails” circling the North Pole



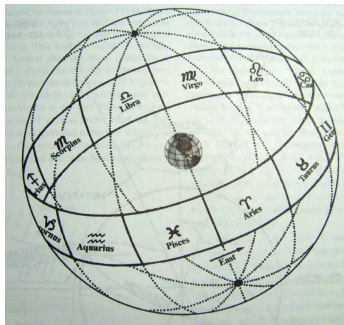
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This time-lapse photograph shows the way the stars seem to revolve overhead when you watch them for awhile on a clear night. The Greeks took this as evidence that the stars are embedded on a rotating Celestial Sphere.

The Sun's apparent motion on the Celestial Sphere

- The Sun goes daily from East to West like the stars.
- And the Sun goes higher in summer and lower in winter—but something less obvious happens, too:
- Watch before dawn to see that the stars near the Sun change a little daily.
- The Sun moves slowly eastward against a band of stars called the *Zodiac*, marking the seasons.
- The great-circle path of the Sun through the middle of the Zodiac is called the *ecliptic*.
- It is only an apparent path—the Earth is doing the moving!

The Zodiac in the Celestial Sphere



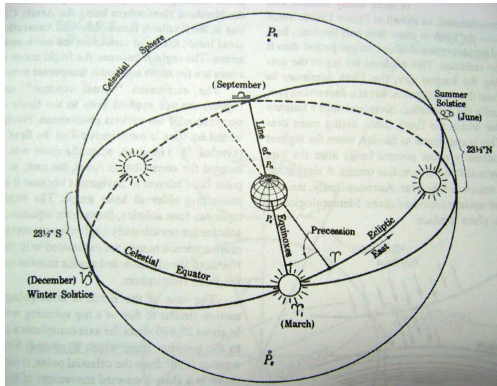
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Here the Earth's axis is portrayed vertically, and the Zodiac is shown inclined. The Greeks saw the Sun moving eastward through the middle of the Zodiac at a rate requiring one year for a full cycle. They supposed that the Celestial Sphere, including the Zodiac along with the Sun, spun from east to west daily around the vertical axis.

A model of the Sun's apparent motion

- We owe the next figure to the ancient Greeks.
- Astronomers use this model today, with modifications.
- It will help us understand the solar analemma.
- It is simple and useful—more so for our purpose than the elliptical orbit shown previously.

Greek model: The Sun orbits the Earth on the ecliptic



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Earth's axis is portrayed vertically, the Sun moves eastward on the inclined *ecliptic* in an annual cycle, and we (not the Greeks) picture the Earth rotating counterclockwise about its axis once every twenty three hours and fifty six minutes. The Earth must spin a little farther than a full revolution to catch up to the moving Sun, which on average over the year takes about four minutes. So the Earth actually spins about 366 times each year—not 365 times!

Discussion of the basic Greek model

The stars are at hugely varying distances from the Earth, but it works well to use the Greek model of stars fixed on a Celestial Sphere equidistant from Earth. The stars move slowly enough compared to their distances from Earth that they appear to maintain fixed separations, and we can use their positions on the Celestial Sphere as a basis for a stable coordinate system. The Celestial Equator (see the previous figure) is then the intersection of the plane of the Earth's equator with the Celestial Sphere. The *ecliptic* (the inclined great-circle path the Sun appears to trace around the Earth) is also the path on which the Sun and Moon must lie in alignment with the Earth for an eclipse to occur.

The astrological signs used in the previous figure to denote the equinoxes and solstices correspond to the constellations of the Zodiac where the Sun was positioned at the time of the Greeks (Aries, Cancer, Libra, and Capricorn), not where they are today. The vernal equinox occurs nowadays when the Sun is "in" Pisces, not Aries, and the very-slow-moving point on the Celestial sphere marking the position of the Sun at the vernal equinox is called *the first point of Aries*, commemorating its location at the time of the Greeks. The symbols for Aries Υ and the first point of Aries Υ_1 appear in the figure. The westward shift of the equinoxes and solstices is due to *the precession of the equinox*, first noted by the Greek astronomer Hipparchus in the second century BC. It is due to a very slow wobble of the Earth's axis, which we now know repeats once every 26,000 years—it is remarkable that Hipparchos detected it. This wobble slowly changes the location of the Celestial Equator on the Celestial Sphere, which is also affected by much smaller deflections of the Earth's axis known only to modern astronomy.

Back to the analemma, this time at Delphi



What causes the analemma?

- The analemma reflects the varying time between successive high noons.
- We have seen that solar (sundial) time varies from day to day for two reasons:
 - (1) the Earth travels at a non-uniform rate around the Sun,
 - (2) the Earth's axis is tilted.
- *Clock time* ticks away at a constant rate marking *mean time*—allocating 24 hours per day, the average length of a solar day.
- The *Equation of Time* plots the difference between clock time and solar time evidenced by the solar analemma.

The Equation of Time resolved into two components

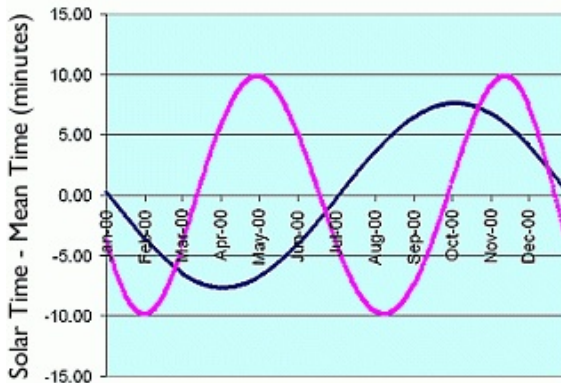
Observe the oddity in the Equation of Time shown on a previous slide in which the solar noon and standard clock noon don't coincide at any canonical time of the year, such as at an equinox or solstice. This curiosity arises because of the way in which the Equation of Time is defined.

The Equation of Time rests on two artificial constructs: (1) the *Intermediate Sun*, and (2) the *Mean Sun*. First we picture the actual Sun moving around the ecliptic at its varying rate, completing an orbit in one year. The Intermediate Sun is a virtual sun that moves around the ecliptic also in one year but at a constant rate. For specificity, the Intermediate Sun is made to coincide with the actual Sun at the time when the Earth is at *perihelion* (the point on the Earth's orbit when the Earth is closest to the Sun, around January 2). The first component of the Equation of Time is then the time difference between the meridian of the actual Sun and the meridian of the Intermediate Sun due to Earth's rotation.

Second, we picture another virtual Sun, the *Mean Sun* running at a constant rate around the Celestial Equator and completing its orbit in one year. For specificity, the Mean Sun is made to coincide with the Intermediate Sun at the time of the Vernal Equinox (around March 22). The second component of the Equation of time is the time of traversal between the meridian of the Intermediate Sun and the meridian of the mean Sun due to Earth's rotation.

The Equation of Time is obtained by adding these two differences, as in the next two slides.

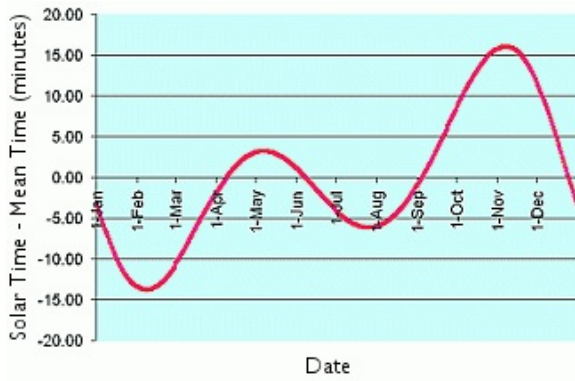
The Equation of Time resolved into two components



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The black curve shows the time difference between the actual Sun and the *Intermediate Sun*, and the red curve shows the time difference between the *Intermediate Sun* and the *Mean Sun*. The Equation of Time is the sum, as explained in the previous slide.

Equation of Time: Difference between solar noon and clock noon



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Note that due to the roundabout way in which the Equation of Time is defined, solar time and clock time don't coincide at any of the canonical times of the year, such as at an equinox, solstice, aphelion or perihelion.

An alternative way to computing the Equation of Time, very briefly

- ϕ = angle of sun on ecliptic from *vernal equinox*
 θ = projection of ϕ on equatorial plane
 ϵ = *obliquity of the ecliptic* (angle of tilt of Earth's axis)
- $\tan \theta = \cos \epsilon \cdot \tan \phi$
- Solutions to $y(t) = 0$ yield clock times at solar noon:
 $y(t) = \tan \theta(t) - \cos \epsilon \cdot \tan \phi(t)$
- Set $t = 0$ at vernal equinox, assume approximate 24-hour intervals, and use Newton's method: $t^{**} = t^* - \frac{y(t^*)}{\dot{y}(t^*)}$
- $\dot{\theta}$ = uniform rotation rate of Earth
 $\dot{\phi}$ = rotation rate of Sun on ecliptic.

The shoulders of giants

- Until Copernicus, astronomers believed that the heavens rotated once daily around a fixed Earth.
- Kepler deduced the Earth travels at a non-constant rate in an elliptical orbit —derived from astronomical measurements by Tycho Brahe.
- These, the Greek astronomers, and Galileo are among the giants on whose shoulders Newton stood.
- Newton showed that elliptic orbits could be explained by an inverse-square law of gravity and laws of mechanics foreshadowed by Borelli, Hooke and Galileo.

Closing thoughts

- Our conception of the solar system and time began with Greek geometry, Copernicus, Kepler, Galileo and Newton's mechanics and calculus.
- Our conception of time has been further refined by Einstein.
- The Global Position System (GPS) would not work without corrections using general relativity.
- We now use quantum mechanics and laser-cooled cesium clocks to measure time.
- Time marches on....

Recommended reading

- Leofranc Holford-Strevens, *The History of Time: A Very Short Introduction*, 2005
- James B. Kaler, *The Ever-Changing Sky: A Guide to the Celestial Sphere*, 2002
- Thomas S. Kuhn, *The Copernican Revolution: Planetary Astronomy in the Development of Western Thought*, 1992
- W. M. Smart, *Textbook on Spherical Astronomy*, 1990

- Also, search with Google, Amazon and in libraries!
- On Internet find Walter Sanford's *Center for Sky Awareness*