Leibniz was the first to understand "e" as base of the natural logarithm and calculate its value accurately: 2.7182818.

First International Virtual Symposium Honoring the "Eulerian" Number e = 2.71828....

February 7, 18:28 Amsterdam time
California 9:28 a.m. pst

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Leibniz solved the Catenary Problem using calculus, BUT he presented it as a geometric construction.

The value of e was needed for the construction.

Based on:

"The Leibniz catenary and approximation of e — an analysis of his unpublished calculations," Hist. Math. (2019),

`https://doi.org/10.1016/j.hm.2019.06.001`,

and presentations at:

`http://mikeraugh.org`.

Catenaries under the Pink Moon

Boston Harbor, April 2017 (mrr)

Upside Down and 631 Feet High

The Gateway Arch, St. Louis, Missouri

Internet

Context

1638, Galileo discussed the hanging chain.

1676, Newton and Leibniz derived the non-constant terms of the exponential series, omitting the constant 1.

1690, Jacob Bernoulli published the challenge.

1691, Leibniz and *Johann* Bernoulli published the first solutions.

1761, J.H. Lambert introduced the hyperbolic functions:

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

Descartes: A curve must be defined by a construction.

Leibniz published a geometric construction, but concealed its discovery by calculus.

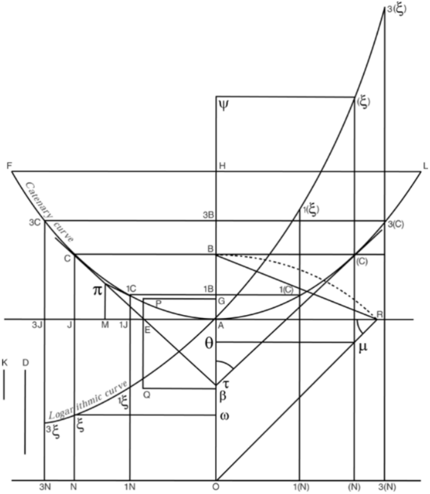
He explained the derivation in a private letter to Rudolph von Bodenhausen but only stated the key ratio 1 : 2.7182818.

"Let those who don't know the new analysis try their luck!"

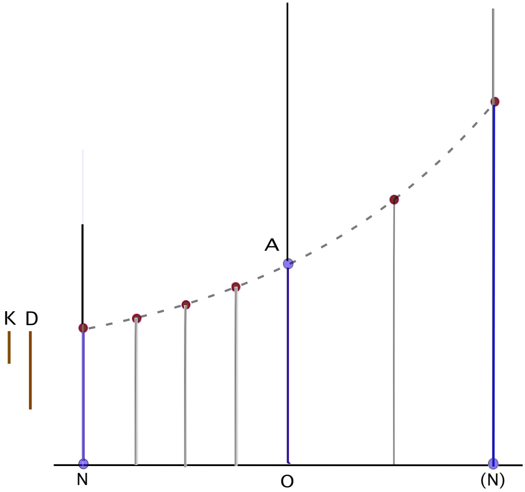
His hidden method of computation was found at the GWLB among his private papers.

And so our story begins with Part 1....Construction

Leibniz's Representation of the Catenary:
Ruler & Compass, K & D

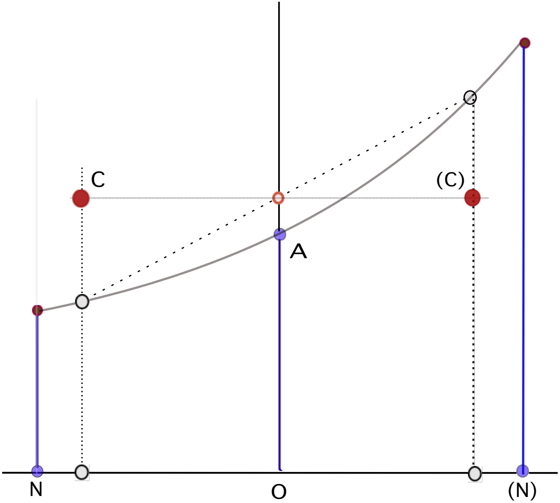


Constructing the "Logarithmic Curve"



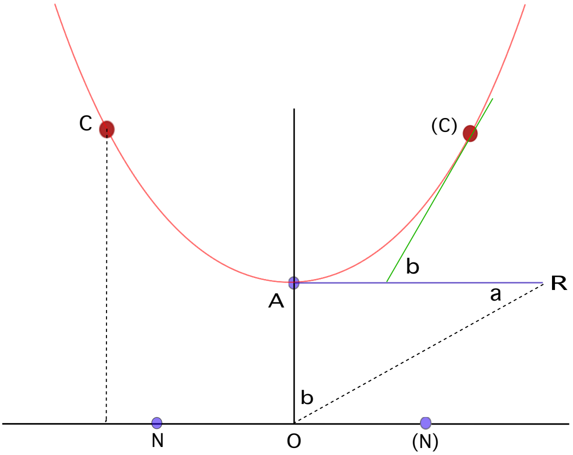
Ordinates over N & O and O & (N) are in ratio K:D.
Middling ordinates are determined by geometric means.

Construction of the "Catenary"



$$C(x) = \frac{r^x + r^{-x}}{2}, \qquad r = \frac{d}{k}$$

Two Examples to Prove that $d/k = e$



Segment $\overline{\text{AR}}$ is equal in length to arc $\widehat{\text{CA}}$.
Tangent at (C) follows from fact that ∠b is the complement of ∠a.
$(y = \cosh x)$

The letter to Von Bodenhausen

He explained how he solved $dy = \frac{d\omega}{\omega}$:

Rudolf Christian Von Bodenhausen, August 1691, with attached Latin text, “Analysis problematis catenarii”, in G. W. Leibniz, Sämtliche Schriften und Briefe, series III, volume 5 (2003), p. 143-155.

He did not say how he found,
2.7182818.

Part 2 will tell that story.....Calculation

Leibniz wrote his calculations on two sides of one sheet

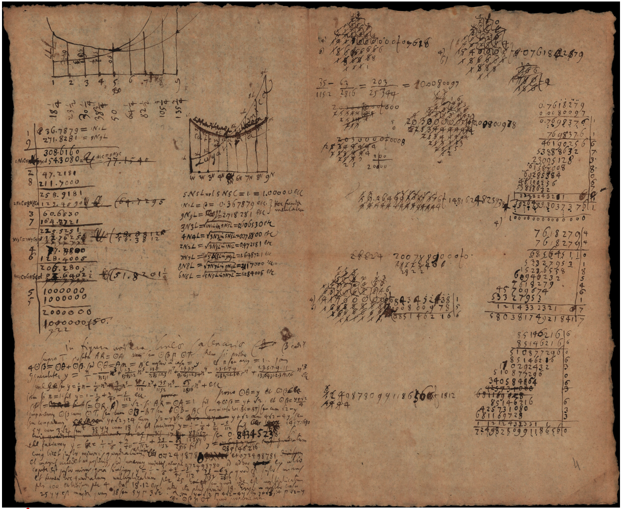
(To see manuscript images courtesy of the GWLB:

http://digitale-sammlungen.gwlb.de/resolve?id=00068056,

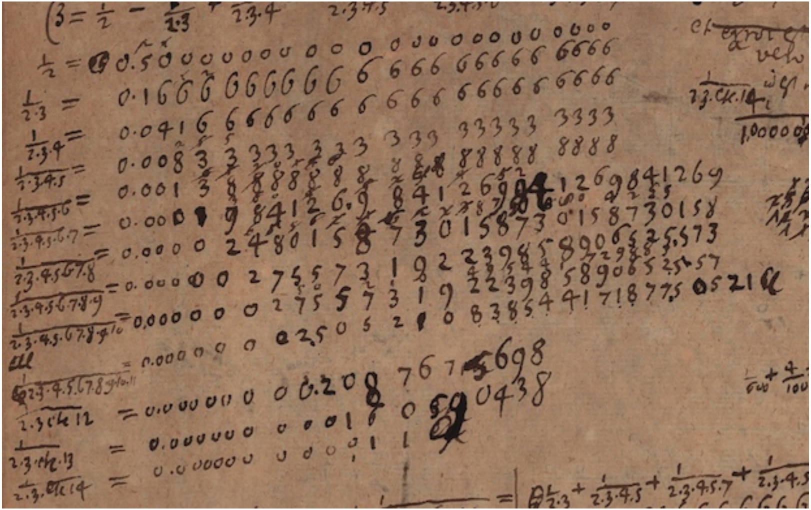
displaying folder LH 35, 6, 11 for holdings relevant to the catenary construcrtion. Click forward on the right arrow until view [5] to see Side 1, and [6] to see Side 2.)

Side 1 of Leibniz's Worksheet: e, $1/e$

Side 2 of Leibniz's Worksheet: Using e, $1/e$



List of Reciprocal Factorials (Side 1)



Reciprocals used to estimate e and $1/e$ (Side 1)

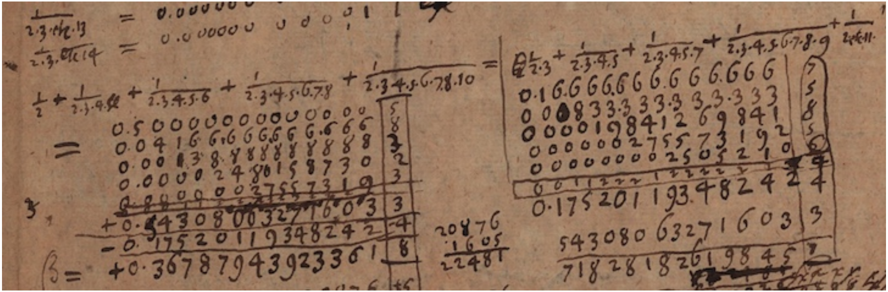


Figure: Reciprocals 1 to 1/11!, even numbers and odd numbers, added or subtracted, yield estimates for e and $1/e$. The digits 20876 and 1605 are misplaced in decimal representations for 1/12! and 1/13!, invalidating attempt to improve results.

HM Fig 5. Leibniz mysteriously corrects his error

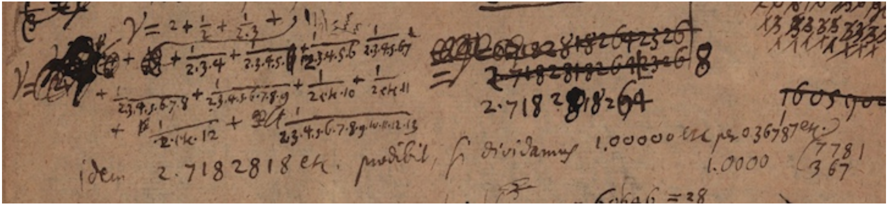


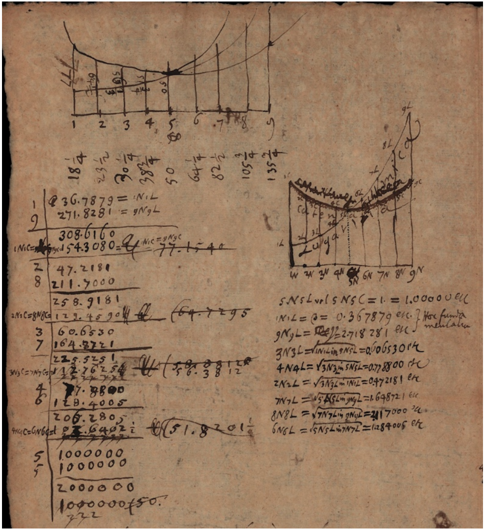
Figure: An incorrect 11-digit estimate for e at right, cut short to a correct 8-digit estimate at left. Why precisely this?

Cumulative Sums (plus +2) Reveal Convergence to e.

Did Leibniz do this?

2)	0.5
3)	0.66666666666666666666666666666667
4)	0.70833333333333333333333333333333
5)	0.71666666666666666666666666666667
6)	0.71805555555555555555555555555556
7)	0.71825396825396825396825396825397
8)	0.7182787698412698412698412698413
9)	0.71828152557319223985890652557319223986
10)	0.71828180114638447971781305114638447972
11)	0.7182818261984928651595318261984928652
12)	0.7182818282861685639463417241195018973
13)	0.71828182844675900231455787011342566898
14)	0.718281828458229747912287594827277367

Construction: the Exponential Curve and Catenary (Side 2)



Drawing and Instructions for the Engraver