THE PROBLEMATIC FOUR BUGS PROBLEM — OR REALITY VS THE CONTINUUM

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ABSTRACT. So my friend Zenau says to me, Mike, have you heard the problem about the four bugs? They start at the vertices of a square mile, and each flies toward the bug counterclockwise from itself. Each flies at a steady rate of 1 mile per hour, each always re-orienting to aim directly at its target bug.

Feeling pretty confident I said, sure Zeneau, the sides shrink at the rate of 1 mile per hour; it must take an hour for all to catch up at the center of the square, assuming they don't butt heads after traveling exactly one mile. This is a well-known problem.

Aah, Zeneau says, that's cute about butting heads, but there's more to this problem than that. I see you don't know what the problem really is.

Zenau can be a pest with questions, but he usually makes a good point. Hmmm, said I, and so this story began....

1. Introduction

Let me tell you Zenau's joke — because, mathematically speaking, it is a kind of joke. I'm afraid I won't tell it in the same crisp way he would do. Let me put it this way:[1]

The bugs have to keep flying at a steady rate, let's say v miles per hour — each toward the one counterclockwise from itself. Assume that the intial gap between them is L_0 miles. So they will close the gaps between themselves in L_0/v hours after traveling L_0 miles. At an initial gap of 1 mile and rate of 1 mph, the bugs would meet after one hour of flight. Seemed like a good-enough argument to me.

There's a similar problem. Two trains are ninety miles apart on a straight track and heading toward one another, the one at 30 miles per hour and the other at 60 mph. A fly starts at the headlight on

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[1]The idea for this note was hatched in conversations with Lewis Mingori at the LACES Calculus Camp in April 2012 and discussed afterwards to the point of exhaustion, a situation comparable to a problem faced by the four bugs. Apropos, I thank Prof. Mingori for pointing out unintended bugs in previous drafts.

one of the engines and flies at 70 mph back and forth between the headlights of the two engines. How far does the fly travel before the trains meet? Assuming the fly has magical powers and can reverse direction instantaneously, then the answer is easy: 70 miles! That's because the trains are closing their ninety-mile gap at 90 mph, so they meet in one hour, and during that time the fly is going 70 mph — ergo the fly flies 70 miles before oblivion. Right? We'll reconsider this problem in the Conclusion.

There is a difference between the two problems: The fly always travels along a straight line, but the bugs do not. So, what kind of path must the bugs follow? And does this make a difference? Let the bugs fly while we answer this question and figure out what Zeneau had in mind.

2. The pursuit path

I reason that, since the bugs start at the corners of a square of side L_0 miles, and each flies at the same constant rate directly toward the counterclockwise bug, they must continue flying in a square formation, and the side of the square must shrink steadily at the constant rate of v miles per hour.[1] (Superscripts hereafter refer to endnotes.)

This implies (does it not) that the bugs will collide at the center in L_0/v hours. If so, the semi-diagonals of the square must shrink steadily too, in fact at the rate of $v/\sqrt{2}$ mph. This is geometry.

So if we let $r(t)$ be the length of a semi-diagonal at time t hours, then we should have,

$$r(t) = r_0 - \frac{v}{\sqrt{2}}t; \quad r_0 = r(0) = \frac{L_0}{\sqrt{2}} \text{ (miles)} \tag{1}$$

Let's make it easy to keep it straight which bug we're talking about. In counterclockwise order, we have Fiery, Moonbeam, Witchcraft and Streak. Let's assume they're Ladybugs, which are actually beetles not bugs, but we're pretending. Let's pay attention to just one of them, the one called Fiery, who will be chasing Moonbeam. We will derive Fiery's pursuit path.

The center of the square will remain fixed during the entire operation. Suppose it's located at the origin. And let the radius vector from the origin to Fiery be $\mathbf{R}(t)$, so that $||\mathbf{R}(t)|| = r(t)$.

Notice that Fiery, in order to always fly directly at Moonbeam, must constantly adjust her direction of flight to make a fixed angle of forty-five degrees to her radius vector from the origin. As we shall see, this is characteristic of the type of flight path Fiery follows. To determine this path, we use polar-coordinate notation for $\mathbf{R}(t)$,

$$\mathbf{R}(t) = r(t)\,(\cos\theta(t), \sin\theta(t))$$

rendering $\mathbf{R}$ in rectangular coordinates, whereby[2]

$$\dot{\mathbf{R}} = \dot{r}\,(\cos\theta, \sin\theta) + r\dot{\theta}\,(-\sin\theta, \cos\theta) \tag{2}$$

Since $\dot{\mathbf{R}}$ is Fiery's velocity vector, and she is flying at the rate of v mph,

$$v^2 = \dot{\mathbf{R}}\cdot\dot{\mathbf{R}} = \dot{r}^2 + r^2\dot{\theta}^2 = \frac{v^2}{2} + r^2\dot{\theta}^2 \tag{3}$$

Therefore, incorporating Eq. (1),

$$\dot{\theta} = \frac{v}{\sqrt{2}}\frac{1}{r} = \frac{v}{r_0\sqrt{2} - vt} \tag{4}$$

But wait! Since this operation is supposed to end at $t = 1/v$ when Fiery and Moonlight supposedly will meet Witchcraft and Streak at the origin, by that time (as evident in Equation (4)) Fiery's radius vector is spinning around the origin infinitely fast! But surely it's impossible for Fiery to make an infinite number of revolutions before completing her one-mile journey! Yet it seems, if she could actually do it, that she will close the one-mile gap between herself and Moonbeam at time $t = 1/v$ — after having spun around the origin an infinite number of times and be infinitely dizzy.

So here is one problem with the Four Bugs Problem, but only the first (and lesser) problem. Let's consider this one for a moment.

Fiery may be small, but she does have mass, say m mils. So consider this. As you can see from Equation (4), during Fiery's flight, her rate of revolution around the origin increases without limit, and at the same time her distance from the origin shrinks to zero. The force she must exert to counteract the increasing outward force to stay on track must also be unlimited. We'll calculate that force in the concluding section (using Newton's $\mathbf{F} = m\mathbf{A} = m\ddot{\mathbf{R}}$), but it's obvious that because she's spinning faster and faster in tighter and tighter loops, that the force she needs to stay on track increases without limit as she approaches the origin. Fiery would have to be fiery indeed!

But, alas, bugs do not have the ability to generate unlimited force. So here's Problem Number 1. These four bugs can't possibly stay on course to finish the operation. They are of small mass, but even so the forces they must resist to stay on course increase to an amount they cannot handle, and they must eventually fly apart. This in itself is a pretty good joke, but no big deal really. Mathematicians can get around a little problem like that.

We just idealize the bugs as points with zero mass. Poof! This converts the problem from one of real bugs of mass m and finite capabilities, to weightless point *models*.[3] We reformulate the problem: if four *points of zero mass* labeled Fiery, Moonbeam, Witchcraft and Streak, move at a constant rate towards one another in the manner specified for the Four Bugs, *then* how long would it take the points to meet at the origin? This purely mathematical problem *should* be easier to solve.

But Zenau is always full of surprises, and Problem 2 lies ahead. Look again at Equation (4); it's inconsistent with the idea that the bugs *meet at the center* after a flight time of $1/v$. The equation for $\dot{\theta}(t)$ blows up at $t = 1/v$, so the domain of $\theta(t)$ is a half-open interval *that excludes* $t = 1/v$. Let us rewrite the equation with this fact made explicit:

$$\dot{\theta} = \frac{v}{\sqrt{2}}\frac{1}{r} = \frac{v}{r_0\sqrt{2} - vt}, \quad 0 \le t < \frac{L_0}{v} \tag{5}$$

This implies that the route that Fiery flies is not defined for $t = L_0/v$. Even the newly branded firebrand Fiery, modeled as a massless point, *can't get to the origin on a path determined by the statement of the problem*. This may be non-intuitive, but there it is! The problem was posed with the implication that the terms defined a solution, but they don't. This is the big problem with the Four Bugs Problem.

It is simply a fact that the condition set for Fiery and her three companions, that they chase directly toward each other, excludes the possibility that they can fly for the full flight time of $T = L_0/v$ hrs. No flight path exists on which the idealized point-bugs can always fly directly toward one another and *also* wind up together at a single point. The most they can do is keep going for time $t \in [0, L_0/v)$, spinning infinitely around the origin without ever reaching the origin. This follows quite directly from Equation (4) and is expressed explicitly in (5). Strictly speaking, the problem is ill posed! (Or at least posed in a misleading way.) The possibility of confusion arose because we were led to think about bugs in the real world actually coming together somewhere, but we wound up having to solve a differential equation, something definitely not in the real world.

This is Zenau's joke, an odd one perhaps, but he's a mathematician. Let's continue the discussion, compare the Four Bugs problem to similar problems in the next section, and see whether there's a way around the problem.

3. Conclusion

It turns out that this famous problem, in which the common answer given is that the four bugs meet at the center after flying for a total time $T = L_0/v$, is not well posed — the solution curves never meet. The problem illustrates the care needed in distinguishing between what can happen in the "real-world" and what a mathematical model may say can happen. The solution involves analysis on the continuum, and that's about as far from reality as you can get.[4]Before getting to some proposed exercises and concluding, let's make some comparisons with two other problems. First, the problem of the fly going back and forth between approaching trains mentioned in the Introduction. The supposition is that the fly can reverse directions instantaneously and maintain a constant speed of 90 mph. A material fly certainly cannot do this, not even once, since it would require infinite force to make each reversal.

So we formulate the problem as a fly modeled as a *point of zero mass*, bouncing back and forth between approaching trains. This is how most mathematicians would think of it. Setting the model within the continuum on an interval I equal in length to the starting distance between the two trains, say $l(I) = D$ miles, the problem implies a set of *turn-around points* on the interval $\{P_i \in I, i = 1, \infty\}$, representing the respective positions of the two trains where the fly meets one of the two approaching trains and reverses motion. The turn-around points are determined by the condition that the respective intervening intervals of lengths d_i (where $d_i = |P_{i+1} - P_i|$) can be traversed at the rate specified for the fly. But note that the point of the interval representing where the trains collide is not a turn-around point.

We can express the total time T of traversal of the "fly" covering the intervals d_i $(i = 1, \infty)$ as

$$T = \sum_{i=1}^{\infty} \frac{d_i}{v} = \frac{D}{R}$$

where, v is the speed of the fly, D is the starting distance between the trains, and R is the combined rate of the two trains approaching each other. We know that the series sums to D/R because the terms $\{d_i/v,\ i = 1, \infty\}$ are times spent by the fly traversing the consecutive turn-around intervals, and hence the partial sums of the series must increase monotonically to the total time allowed for the fly's flight, namely the time it takes the two trains to meet: D/R. Therefore, we can exercise the privilege of mathematicians and *define* the "distance the fly travels" by

$$S := \sum_{i=1}^{\infty} d_i = v \cdot \frac{D}{R} \tag{6}$$

This mathematical formulation of the problem allows you to say that the infinite series implied by the fly's turn-around points converges to vD/R, but it does not allow you to conclude that the fly actually winds up at the collision point, although you may say so with a wink. Plugging in the data given in the Introduction ($D = 90$, $v = 70$ and $R = 90$), we get the result for that case: $S = 70$.[5]

The solution is sophisticated because it rests on the theory of infinite series and thereby the continuum. Moreover, depending on how you read the theory, you can conclude the fly never actually travels the full distance given by the sum of the infinite series, since none of the turnaround points is at the collision point — and the fly can reach only to distances expressed by partial sums of the infinite series in Eq. (6). Or you can say in terms of mathematical convention that the fly travels a full distance equal to the limiting sum of the series. But then we're talking about mathematical conventions and points we call a "fly," not about trains and flies.

Lets compare this with the famous *Achilles Paradox* of Zeno. Achilles, according to myth, was the fastest runner in ancient Greece. He chased a tortoise but could never catch up to it because, first he must reach the point where the tortoise started, at which time the tortoise will have advanced and Achilles must then run to that point, and so forth. Since this process of catching up must go on infinitely often, and it is impossible for a man to do anything infinitely often, Zeno concludes it must be impossible for Achilles to ever catch up.

Here again we encounter a categorical difference between the "real world" in which a supposed person (Achilles) chases after a live tortoise, and the mathematical continuum. To help make the distinction clear, suppose you pencil a line segment with Achilles and the tortoise in their starting positions at the endpoints. Then extend the line segment to where the tortoise gets at the time Achilles arrives at the starting point for the tortoise. Then repeat the marking of tortoise points that thereafter are reached in sequence by Achilles. We know from real-world experience that Achilles will catch up to the tortoise after the tortoise reaches a short distance farther away on the extended line. So there seems like there must be an infinity of points crowded ever more closely to the point where Achilles catches up. But what does that mean? The pencil point is too fat for you to mark all the points, and you don't have infinite time for it anyway. Neither is there a machine in the world that can pick out a perfect point on a perfect line or have time to mark an infinite number of perfectly distinct points. Or if there are such things, I sure don't know about them.

So where does this *infinity* come from? Even in set theory, we need an *axiom* for infinity, we can't just assert that there *is* an infinity.[6]

Zeno puts Achilles and the tortoise into the realm of pure numbers (to us, the continuum) and thereby confounds reality with an abstract mathematical model of a line. But we can resolve this in modern terms by starting with the assumption that although the original problem is postulated in the real world, we will model it mathematically as a problem in analysis.

Imagine Achilles as a function that relates positions on the real line to *time* $t \geq 0$ represented as a real number, such as $A(t) = vt$, where v is the running speed of the "person" Achilles. To illustrate how abstract this simple model is, note that the function representing Achilles is in fact a set of ordered pairs: given $v \in \mathbb{R}_{>0}$, $A_v := \{(t, A(t)) \mid t \in \mathbb{R}_{\geq 0}, A(t) = vt\}$. In this mathematical world, the function "Achilles" covers a continuum of points beyond the origin for any $t > 0$. If the turtle T moves at rate $\phi > 0$ mph and starts at a distance $d > 0$ from Achilles, and we model the turtle as $T_\phi := \{(t, T(t)) \mid t \in \mathbb{R}_{\geq 0}, T(t) = d + \phi t\}$, then Achilles will meet the turtle at time $t = d/(v - \phi)$. Not a paradox in this mathematical idealization, and not a perfect description of reality either.[7] This solution of Zeno's Paradox, in which the continuum and methods of analysis are used, is known by philosophers as the *Standard Solution*.[8]

Something like the situations for Achilles and the fly is true for the four bugs. No material bug can fly all the way along the tighter loops required by the problem, and the four ideal curves defined by the terms of the problem (supposing the bugs are simply four points traveling toward one another at a constant speed) never meet, although the four curves do *converge* to the origin. So the two requirements that the four bugs *stay on course* like mobile points on a curve and *meet at the origin* are incompatible. The problem is posed in a way that misleads the unwary, because a problem expressed in terms of real-world behavior is confounded with a problem in differential geometry. The problems faced by Achilles, the fly and the four bugs are examples showing the importance of keeping clear the distinction between reality and the continuum when applying mathematics to "real" problems.

For fun, let's return to the problem of dynamics. You might differentiate Equation (2) to find the precise acceleration Fiery undergoes while staying on track toward Moonbeam.[9] Her pursuit path is a spiral obtained by integrating the Equation (4), yielding the parametric equations,

$$r(t) = r_0 - \frac{v}{\sqrt{2}}t; \quad \theta(t) = \ln(L_0 - vt); \quad 0 \le t < L_0/v \tag{7}$$

with limiting value,

$$\lim r(t) = 0 \text{ as } t \to \frac{L_0}{v}$$

This type of spiral is known as a *logarithmic* spiral. But, because Fiery's spiral flight makes a fixed angle with her radius vector from the origin, it is also called an *equi-angular* spiral.[10] Facts about this Interesting class of curves can be found on the Internet.

Here's another thing you might want to examine for yourself. Logic has implied that Fiery's planned flight path is just one mile in length. .But is that actually the length of the spiral derived above? With the caveat that the path is missing an endpoint, you can see that this is so by looking at the first two members of Equation (3). This means that Fiery's equi-angular spiral, which spirals around the origin infinitely often is actually finite in length; more precisely, the distance Fiery travels *approaches* one mile *as a limit* as her time of flight approaches $t = L_0/v$.

Another good exercise is to check consistency. At the outset, we assumed that because the four bugs adjust their flight direction to stay pointed directly at the counter-clockwise bug, that their square formation would shrink at exactly their speed of flight. Now that we have derived an equation for Fiery's flight path we can check the consistency of our assumption with the result. Prove that bugs traveling on "parallel" paths (i.e., paths rotated ninety degrees from the path of the clockwise bug) like the one for Fiery parametrized in Equation (7) do actually satisfy the conditions of the problem: the four bugs remain at the corners of a spiralling shrinking square, and each one's direction of travel always points directly at the bug counterclockwise to itself.[11]

Since we've been discussing applications of mathematics to "real-world" problems, we should try to find something practical in this. Some night-flying insects navigate by maintaining afixed angle of flight with reference to the Moon. This is why they spiral futilely around an electric light or crash into a candle flame. Similarly, a plane always traveling at a fixed bearing (not due North or South) will spiral around a pole in a three-dimensional curve called a *loxodrome*.

Zenau's problem illustrates subtleties lurking in mathematical modeling of physical phenomena, even in simple puzzles.

4. Afterword

After writing the piece above I recalled a book that had been sitting on my shelf for many years, the fourth edition of Tobias Dantzig's *Numbers: The Language of Science*.[12] I had bought a copy after meeting Dantzig when I was in high school, but layed it aside. The book begins with some history of ancient symbols and methods of numeration, which at that age didn't interest me. I wanted to learn more about calculus, not how shepherds counted flocks. So, without delving further, I let the book sit on my shelf for all these intervening years.

What a pity! Dantzig shows what a stupendous intellectual achievement it was to go from tabulating sheep to the theory of the number system required for modern analysis. This is a theme I attempted to underscore in the article above! Better late than never.

What can be more interesting than origins, the tracing out of processes by which humankind has gained knowledge? Dantzig's Part I tells the story: "The Evolution of the Number Concept." In the last chapter of Part I he presents an outline of "Milestones in the Evolution in the Number Concept" from antiquity through the Nineteenth Century.

Some of Dantzig's offhand comments have been superannuated by later research; he does not recognize that primates have a number sense. But this is beside the point. His telling of mathematical developments is sure footed. Without knowing of Robinson's later model of the continuum that includes differentials and infinity among the real numbers, Dantzig allows that future theories may extend the real numbers. In explaining how the continuum created a construct for the continuous flow of time, Dantzig also quotes Hilbert on how the quantum theory denies continuity in nature. There are deep philosophical issues here with matching mathematics.

Dantzig's book remains a thoughtful popular introduction to quandries raised and resolved by the number system. "This is beyond doubt the most interesting book on the evolution of mathematics which has ever fallen into my hands." (Albert Einstein, quoted on the book's cover)

Further reading is suggested by Mary Tiles cited in the endnotes.

Notes

[1] This assertion one can doubt, but it's plausible because by symmetry you can be pretty sure the bugs continue flight at the corners of a square, and at the instant one bug flies directly at the counterclockwise one, that one is instantaneously heading at a right angle toward the bug counterclockwise to *her*. Thus there is no component of velocity other than in the direction of the counterclockwise bug, so the shrink rate of the square must be the full v mph. You can confirm that the mathematical model that follows reflects this expectation ex post facto, as suggested among the problems posed in the Conclusion.

[2] About notation. Uppercase boldface letters are used to represent vectors, the *dot* notation between two vectors indicates the *scalar* product of the two vectors, and $||\mathbf{A}||$ represents the *magnitude* (length) of $\mathbf{A}$. Note that our representation of vectors uses polar coordinates (ρ, θ) and uses them to represent the vector as a pair of Euclidean coordinates: $(\rho\cos\theta, \rho\sin\theta)$. For unit vectors $\mathbf{A}$ and $\mathbf{B}$, it is always true that $\mathbf{A}\cdot\mathbf{B} = \cos\theta$, where θ is the angle between the two vectors. For a particle with position on a curve given parametrically as a function of time, for example $\mathbf{R}(t)$, $\dot{\mathbf{R}}$ is the *velocity vector* of the particle at time t, and the *speed* of the particle is given by $v = ||\dot{\mathbf{R}}|| = \sqrt{\dot{\mathbf{R}}\cdot\dot{\mathbf{R}}}$.; if the curve is smooth, the velocity vector is always tangent to the curve, and $\ddot{\mathbf{R}}$ is the acceleration vector of the particle at time t.

[3] Here's Wikipedia: "A mathematical model is a description of a system using mathematical concepts and language." If the system is something in the real world, the question one should ask is: How well does the model describe the system? Pretty good, as it turns out, when NASA uses analytical mechanics to model satellite trajectories. Maybe not so good when Wall Street uses mathematics to model risk.

[4] I use the term *continuum* as a synonym for the *real number system* to emphasize its special character. The continuum is a modern invention, arising from the work of Cauchy, Weierstrass, Dedekind, Cantor, Whitehead and Russell, inter alia. The continuum has roots in the theory of sets, and it has many mysteries. For example, the continuum is uncountable, but there can only be a countable number of algorithms, so most numbers can never be individually specified. If they can't be specified, what good are they? Plenty good, as it turns out, since by allowing for the existence of limits they make analysis possible.

For a development of the real numbers within the context of set theory, see Yiannis Moschovakis' *Notes on Set Theory, 2nd ed*, Springer 2006.

See also the preface and first five chapters of *Analysis I, Vol 1, 2nd edition*, by Terence Tao, Hindustan Book Agency, November 12, 2009. That Tao would take such care in five chapters of an undergraduate honors course to develop the real number system is in itself an indication of the importance and subtlety of the subject. (Having seen the Amazon Reviews, I want to say that I like Tao's leisurely explanatory style, which reveals more of the underlying thought than, for example, the arid brevity of Rudin's *Real Analysis*, which obscures motivation although it is good for reference and review.)

A remarkable "novel in cartoons" tells the story of the founding of modern mathematical logic and set theory, including as subplot the struggle to understand the continuum: *Logicomix — An Epic Search for Truth* by Apostolos Doxiadis, Christos H. Papadimitriou, Alecos Papadatos and Annie Di Donna, Bloomsbury

USA; September 29, 2009. I thank mathematician Roja Bandari for this surprising reference. Can you believe it that a book with such a recondite topic could become a NYT bestseller? (See the Amazon Book Description and Reviews.)

[5] For given parameters (D, v, R) you can certainly derive the turn-around points in terms of explicit train positions, then sum the resulting geometric series as allegedly John Von Neumann did very quickly for some version of the problem, but that level of detail is unnecessary.

[6] Take a moment to consider the difference between how easy it is to *imagine* a penciled line, and how hard it is to *construct* the continuum in a mathematically rigorous way. The Greeks had enough trouble dealing with "irrational" numbers, and never got far enough to conceive of a rigorous notion of the real number system, and Zeno's Paradox reveals how much trouble they had thinking about a denumerable infinity.

[7] I have expressed this in a strictly abstract way to emphasize the set-theoretic nature of the functions that define the "motions" of Achilles and the Tortoise, and their dependency on the continuum ($\mathbb{R}$).

[8] See "Zeno's Paradoxes" by Bradley Dowden in the *Internet Encyclopedia of Philosophy*, article last dated Apr 1, 2010. For more philosophical discussion about the logic and set theory used in developing the theory of the continuum, and about Zeno's paradoxes in particular, see *The Philosophy of Set Theory: An Historical Introduction to Cantor's Paradise*, by Mary Tiles, Dover Books on Mathematics, 2004. See especially Chapter 4, "Numbering the Continuum," for a discussion of ways Cantor used to come to grips with the strangeness of the continuum.

[9]Since v^2 is constant, it follows from Equation (3) by differentiation that $\ddot{\mathbf{R}} \cdot \dot{\mathbf{R}} = 0$. This shows that Fiery's acceleration $\ddot{\mathbf{R}}$ is normal to her flight path. So none of the force she applies in flight is used to maintain speed. (We're ignoring friction.) All her strength is applied to staying on course. Assuming Fiery has mass $m > 0$, she must resist a force of

$$m||\ddot{\mathbf{R}}(t)|| = \frac{mv^2}{L_0 - vt}$$

normal to her direction of flight, which approaches infinity as t approaches L_0/v.

[10] It was remarked in passing that Fiery must always fly at an angle of forty-five degrees to her vector from the origin. And a previous endnote mentioned the fact that the dot product between two unit vectors equals the *cosine* of the angle between them. Doesn't this imply that the following should be true for the path $\mathbf{R}(t)$ that we derived?

$$\frac{\dot{\mathbf{R}} \cdot \mathbf{R}}{||\dot{\mathbf{R}}||\,||\mathbf{R}||} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \tag{8}$$

Work it out to see if this is correct. Why isn't it?

[11] Check this for Moonbeam by noting that the vector of length $L_0 - vt$ in the direction of Fiery is $(L_0 - vt)\dot{\mathbf{R}}(t)/v$. Add this to $\mathbf{R}(t)$ to see that the resulting vector terminates where Moonbeam should be — on the same kind of spiral as Fiery's, rotated counterclockwise ninety degrees. (Use Equations (2), (3) and (4).) From this you can also see that at any instant, Fiery's aim at Moonbeam is directly perpendicular to the direction of Moonbeam's flight at that instant.

[12] Tobias Dantzig, *Numbers: The Language of Science. Fourth Edition, Revised and Augmented*, The Free Press, 1954.